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13. ABSTRACT (*Maximum 200 words*)

There were a number of significant accomplishments during the four years. During this period, 21 papers were published, 5 preprints were written, and 24 invited lectures were presented. Studies of nonlinear waves in thin film ferromagnets have shown the nonlinear Schrödinger equation (NLS) to be a fundamental equation. A first principles derivation of the NLS equation in concrete situations has been completed. The derivation together with numerical computations are being used to compare with actual experimental data generated at Colorado State University. The current analysis complements previous results involving nonlinear wave propagation in bulk ferromagnets where a generalized Kadomtsev-Petviashvili equation was derived. Recent research in quadratically nonlinear optical materials in multidimensions has demonstrated that coupled NLS type equations govern quasi-monochromatic wavetrains. The equations couple the optical field to DC fields. Ferromagnetic systems under study are quadratically nonlinear and therefore it is expected that similar coupled systems will govern slowly varying multidimensional wave trains.

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Nonlinear Waves, Optical Soliton Propagation and Computation

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Final Report
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OBJECTIVES

To conduct fundamental research involving nonlinear electromagnetic wave propagation involving nonlinear optics and ferromagnetic phenomena.

STATUS OF EFFORT

Studies of nonlinear waves in thin film ferromagnets have shown the nonlinear Schrödinger equation (NLS) to be a fundamental equation. A first principles derivation of the NLS equation in concrete situations has been completed. The derivation together with numerical computations are being used to compare with actual experimental data generated at Colorado State University. The current analysis complements previous results involving nonlinear wave propagation in bulk ferromagnets where a generalized Kadomtsev-Petviashvili equation was derived. Recent research in quadratically nonlinear optical materials in multidimensions has demonstrated that coupled NLS type equations govern quasi-monochromatic wavetrains. The equations couple the optical field to DC fields. Ferromagnetic systems under study are quadratically nonlinear and therefore it is expected that similar coupled systems will govern slowly varying mutidimensional wave trains.

ACCOMPLISHMENTS/NEW FINDINGS

i) Nonlinear Waves in Thin Film Ferromagnets

It is well known that magnetic systems support nonlinear wave propagation. Particular interest has centered on the properties of nonlinear waves in bulk and thin film ferromagnets.

Experiments in Professor Carl Patton's group in the Physics Department at Colorado State University have motivated our studies. In these experiments, solitons and multisolitons are produced as follows. An yttrium iron garnet (YIG) thin film is placed in a saturating magnetic field. This causes the dipoles of the ferromagnet to align with the field. A microwave signal is then introduced into the thin film at one end using a transducer strip. This microwave signal perturbs the dipoles which induces precession. Neighboring dipoles form a propagating envelope spin wave. When the spin wave reaches the far end of the thin film, a second transducer strip is used to obtain an output waveform. Soliton type waves have been observed. The number of solitons produced in an experiment depends on the power of the microwave signal. The nonlinear Schrödinger equation (NLS) governs the propagation of these solitons for certain geometries.

Our research involves developing a first principles theory to model nonlinear wave propagation in thin film ferromagnets. Since the Colorado State experiments are conducted in thin films with small transverse dimension, one dimensional wave propagation provides an adequate description. In these ferromagnets, there are three distinct cases depending on the direction of the saturating magnetic field. Suppose the thin film is perpendicular to the z-direction and the propagation of the magnetic envelope spin waves to be in the x-direction. The direction transverse to x,z is the y-direction. So called magnetostatic forward volume waves (MSFVW) are formed if the saturating magnetic field is orthogonal to the film surface and the direction of propagation. Magnetostatic backward volume waves (MSBVW) are produced if the saturating field is parallel to both the surface of the thin film and the direction of propagation; magnetostatic surface waves (MSSW) are created if the saturating field is parallel to the thin film surface and orthogonal to the propagation. These cases are named according to properties of the linear wave propagation in the thin film. Since these waves propagate much slower than the speed of light in the thin film, we assume the magnetostatic approximation of Maxwell's equations, hence the term magnetostatic waves.

The NLS equation based on a heuristic derivation has been used for a number of years to describe the nonlinear wave propagation in ferromagnetic thin films. The coefficients in the equation are therefore not fixed by a rigorous analysis. Our approach is to use multiple scales perturbation theory to derive the governing NLS equation.

In the case of MSFVW, we have compared solutions obtained from our NLS equation to

experiments by the Colorado State group through the use of numerical computation with the integrable discrete nonlinear Schrödinger equation (IDNLS). Runge-Kutta and predictor-corrector methods are implemented to time integrate IDNLS. Other researchers in Patton's group have also used the IDNLS equation successfully in their research. The NLS equation appears to predict the formation of 1-2 solitons well. Higher order solitons are obtained at high power; for this case other perturbative effects need to be incorporated in the model.

A future goal is to produce consistent and repeatable periodic soliton wavetrains in order to construct a device such as a soliton oscillator. This device has potential application in radar delay lines which has defense and civilian relevance.

In addition, we are considering thin films with a larger transverse dimension (y-dimension). We are developing a first principles theory for two dimensional nonlinear wave propagation. Preliminary results indicate that the governing equations are a $(2 + 1)$ dimensional generalization of the NLS equation—namely a coupled system where the magnetic field is coupled to the mean term. We refer to these equations has NLSM (i.e. NLS equations with a mean term). This is analogous to the well-known Benney-Roskes/Davey-Stewartson equation which arises in water waves.

In fact, we have recently obtained such nonlinear coupled systems in nonlinear optics where DS has been found to govern nonlinear waves in $\chi^{(2)}$ materials (see below). At present, however, we cannot compare these NLSM equations to experiments since only relatively narrow films have been used. Here narrow means that the film's transverse dimension is much smaller than the slow modulation in the longitudinal direction. This narrowness essentially limits the propagation to a 1 dimensional waveguide. In the future, our colleagues at Colorado State hope to experimentally investigate such two dimensional "wafer" thin films.

ii) Nonlinear Waves in Bulk Ferromagnets

In addition to our thin film research, we have considered nonlinear wave propagation in bulk (e.g. thick films) ferromagnets. In this problem, Maxwell's equation is considered but without the magneto-static approximation. The usual torque equation relates the magnetization and magnetic fields, analogous to the way the polarization and electromagnetic fields are related in nonlinear optics. In the long wave approximation with waves propagating in the x-direction, and assuming slow transverse (in the y-direction) variations, a nonlocal multidimensional nonlinear wave equation is derived. Important in this derivation is a conservation law: conservation of the magnitude of the magnetization, which follows from the torque equation. An approximation of the nonlocal equation reduces to a generalized Kadomtsev-Petviashvili equation (GKP) for suitably small amplitude waves. This nonintegrable GKP equation has been studied by Ablowitz, Wang, and Segur. They found for

suitable choices of sign that there is wave collapse This suggests the possibility of significant pulse compression in this case.

iii) Multi-dimensional pulse propagation in quadratic optical materials

It is well known that the nonlinear Schrödinger equation (NLS) is a centrally important equation in nonlinear optics, water waves, plasma physics, etc. In optics, the NLS equation is used to explain a variety of phenomena such as soliton communication, optical switching, collapse phenomena (and related eye damage due to collapse), etc.

In many optical applications, the underlying nonlinearity is quadratic ("$\chi^{(2)}$" materials). In multidimensions, we find that the nonlinear equations in $\chi^{(2)}$ materials governing quasi-monochromatic wave trains is not the usual NLS equation but rather a coupled nonlinear system involving both the optical field and mean terms. We call these equations NLSM systems. Indeed, in water waves similar systems have been derived. As mentioned above, they were first found by Benney and Roskes in 1968. A few years later a special case of this system was found to be integrable. The latter system is often referred to as the Davey-Stewartson (DS) system.

In the optical application, we have derived both scalar and vector NLSM systems. The vector NLSM systems generalize to multidimensions the well known 1+1 vector NLS equations. The vector multidimensional system have no analog in water waves or plasma physics. The scalar systems are similar in structure to the equations derived in water waves. In this regard, one finds that one dimensional solitons (i.e. NLS solitons) are unstable to slow transverse perturbations. In the integrable case, stable localized pulses which are driven by the associated mean fields are known to exist. We believe that similar pulses will exist even in the more general non-integrable case which we derive in optics. Numerical simulations are currently being employed to study this situation. We are collaborating with Steve Blair who is a Ph.D. student in Kelvin Wagner's research group. Blair believes that these multi-dimensional pulses may be observable in the laboratory. This would be the first time such stable localized multi-dimensional pulses have been observed.

Potential applications include terrahertz imaging and optical switching. Both applications have important civilian and defense applications.

PERSONNEL SUPPORTED

- Faculty: Mark J. Ablowitz (nonsupported)
- Post-Doctoral Associates:

  None

- Graduate Students:

  Scott E. Mock

  Rudy Horne

- Other (please list role)

  None

PUBLICATIONS

- **SUBMITTED**
  - **Books/Book Chapters**
  - **Journals**
    1. Multi-dimensional Pulse Propagation in Non-resonant $\chi^{(2)}$ Materials, M.J. Ablowitz, G. Biondini and S. Blair, APPM† #313 (May 1997), submitted *Opt. Lett.*
  - **Conferences**

  APPM†: Department of Applied Mathematics report

- **ACCEPTED**
  - **Books/Book Chapters**
    1. Numerical Stochasticity, Hamiltonian Integrators and the Nonlinear Schrödinger Equation, M.J. Ablowitz and C. Schober, in *Three Dimensional Dynamical Systems,* Ed. Dr. Kandrup, Annals New York Academy of Sciences, (1994) 162-181.
    2. Discretizations, Integrable Systems and Computation, APPM† #306 (November 1996), (to appear in *Important Developments in Soliton Theory–II*).
    3. Chaos in Numerics, B.M. Herbst, G.J. Le Roux and M.J. Ablowitz, in *Numerical Analysis,* Eds. D.F. Griffiths and G.A. Watson, World Scientific, Singapore, (1996) 133-154.
  - **Journals**
    1. Solutions to the 2+1 Toda Equation, J. Villarroel and M.J. Ablowitz, *J. Phys. A.*, **27** (1994) 931-941.

2. Wave Collapse and Instability of Solitary Waves of a Generalized Nonlinear Kadomtsev-Petviashvili Equation, X.P. Wang, M.J. Ablowitz, and H. Segur, *Physica D,* **78** (1994) 241-265.
3. Homoclinic Manifolds and Numerical Chaos in the Nonlinear Schrödinger Equation, M.J. Ablowitz and C. Schober, *Mathematics and Computers in Simulation,* **37** (1994) 249-264.
4. Effective Chaos in the Nonlinear Schrödinger Equation, M.J. Ablowitz and C. Schober, *Contemporary Mathematics,* **172** (1994) 253-268.
5. Multisoliton Interactions and Wavelength-Division-Multiplexing, S. Chakravarty, M.J. Ablowitz, J.R. Sauer, R.B. Jenkins, *Opt. Lett.,* **20** (1995) 136-138.
6. Integrability, Computation and Applications, M.J. Ablowitz, S. Chakravarty and B.M. Herbst, *Acta Applicande Mathematicae,* **39** (1995) 5-37.
7. Data-Dependent Timing Jitter in WDMS Soliton Systems, R.B. Jenkins, J.R. Sauer, S. Chakravarty and M.J. Ablowitz, *Opt. Lett.,* **20** (1995) 1964-1966.
8. Numerical Simulation of Quasi-Periodic Solutions of the Sine-Gordon Equation, M.J. Ablowitz, B.M. Herbst and C.M. Schober, *Physica D,* **87** (1995) 37-47.
9. Computational Chaos in the Nonlinear Schrödinger Equation Without Homoclinic Crossings, M.J. Ablowitz, B.M. Herbst and C.M. Schober, *Physica A,* **228** (1996) 212-235.
10. On the Numerical Solution of the Sine-Gordon Equation I. Integrable Discretizations and Homoclinic Manifolds, M.J. Ablowitz, B.M. Herbst and C.M. Schober, *J. Comp. Phys.,* **126** (1996) 299-314.
11. The Burgers Equation Under Deterministic and Stochastic Forcing, M.J. Ablowitz and S. De Lillo, *Physica D,* **92** (1996) 245-259.
12. Four-wave Mixing in Wavelength-division Multiplexed Soliton Systems–Damping and Amplification, M.J. Ablowitz, G. Biondini, S. Chakravarty, R.B. Jenkins and J.R. Sauer, *Optics Letters,* **21** (1996) 1646-1648.
13. Solutions to the Time Dependent Schrödinger and the Kadomtsev-Petviashvili Equations, M.J. Ablowitz and J. Villarroel, *Phys. Rev. Lett.,* **78** (1997) 570-573.
14. The Nonlinear Schrödinger Equation: Asymmetric Perturbations, Traveling Waves and Chaotic Structures, M.J. Ablowitz, B.M. Herbst and C.M. Schober, *Mathematics and Computers in Simulation,* **43** (1997) 3-12.

15. Initial Time Layers and Kadomtsev-Petviashvili Type Equations, M.J. Ablowitz and X-P. Wang, *Stud. Appl. Math.*, **98** (1997) 121-137.
16. On the Numerical Solution of the Sine-Gordon Equation II. Performance of Numerical Schemes, M.J. Ablowitz, B.M. Herbst and C.M. Schober, *J. of Comp. Phys.*, **131** (1997) 354-367.
17. Four Wave Mixing in Wavelength-division Multiplexed Soliton Systems: Ideal Fibers, M.J. Ablowitz, G. Biondini, S. Chakravarty, R.B. Jenkins, and J.R. Sauer, *J. Optical Soc. of America B,* **14** (1997) 1788-1794.

– **Conferences**

1. Hamiltonian Integrators for the Nonlinear Schrödinger Equation, M.J. Ablowitz and C. Schober, in *Proceedings of the 2nd IMACS Conference on Computational Physics,* Ed. J. Potvin, World Scientific, Singapore, (1994) 219-224.
2. On a New Class of Lump Type Solutions to the Kadomtsev-Petviashvili and Nonstationary Schrödinger Equations, M.J. Ablowitz, APPM† #305 (November 1996) (to appear in *Proceedings, Advances in soliton theory and its applications—The 30th anniversary of the Toda lattice, Yokohama, Japan*).
3. New Solutions of the Nonstationary Schrödinger and Kadomtsev-Petviashvili Equations, M.J. Ablowitz and J. Villarroel, APPM† #308 (December 1996), (to be published in Proceedings SIDE II Conference, Canterbury, UK, 1996, Cambridge Univ. Press).
4. On the Numerics of Integrable Discretizations, M.J. Ablowitz, B.M. Herbst and C.M. Schober, in *CRM Proceedings and Lecture Notes,* Centre de Recherches Mathematiques, **9** (1996) 1-11.
5. Dynamics of Multi-phase Solutions of a Perturbed Nonlinear Schrödinger Equation, M.J. Ablowitz and C.M. Schober, APPM† #318 (May 1997), to appear *Proc. IMACS Conf.*.

INTERACTIONS/TRANSITIONS

- Participation/Presentations At Meetings, Conferences, Seminars, Etc (M.J. Ablowitz)

1. Nonlinear Optics and Communications Workshop, Breckenridge, Colorado, "Multisoliton Interactions in Nonlinear Optical Fibers", April 11-12, 1994.
2. Symmetries and Integrability of Difference Equations, Montreal, Canada, "Computational Chaos in Integrable Systems–Truncation and Roundoff", May 23-24, 1994.

3. AFOSR Meeting on Computational and Physical Mathematics, Kirtland AFB, New Mexico, "Numerical Chaos in Coherent Systems", June 1-3, 1994.
4. Workshop on Twenty Years of the Nonlinear Schrödinger Equation and Recent Developments, Moscow, Russia, "Computational Chaos in the Nonlinear Schrödinger Equation", July 23-31, 1994.
5. University of Alberta, Edmonton, Alberta, "Computational Chaos in Integrable Systems", Sept. 8-10, 1994.
6. NEEDS Workshop, Los Alamos National Laboratory, Los Alamos, NM, "Integrability, Computation and Nonlinear Optics", Sept. 11-14, 1994.
7. Nonlinear Optics Workshop, Dept. of Mathematics, University of Arizona, "Soliton Interaction in Nonlinear Optics", October 9-11, 1994.
8. University of Tokyo, Applied Mathematics Department, "Novel Integrable Systems", Nov. 9, 1994.
9. University of Science and Technology, Hong Kong, "Computational Chaos in Integrable Systems", Nov. 16, 1994.
10. Hong Kong Polytechnic, Hong Kong, "Computational Chaos in Integrable Systems", Nov. 17, 1994.
11. Colorado State University, Dept. of Mathematics, "1895-1995: Integrability and Applications", March 30, 1995.
12. PhD Course, KdV'95, University of Amsterdam, the Netherlands, "Numerical Computation of Integrable Systems I and II", April 21, 1995.
13. International Symposium, KdV'95, Amsterdam, the Netherlands, "1895-1995: Integrability and Applications", April 25, 1995.
14. Mathematics Department, University of Leeds, United Kingdom, "1895-1995: Integrability and Applications", April 27, 1995.
15. Mathematics Department, University of Loughborough, United Kingdom, "1895-1995: Integrability and Applications", May 5, 1995.
16. Workshop on Nonlinear Optics, Mathematics Department, University of Arizona, "Nonlinear Schrödinger Equations and Wavelength Division Multiplexing", October 2, 1995.
17. Department of Mathematics, Kent University, Canterbury, England "Computational and Effective Chaos in Integrable Systems", November 17,1995.

18. Conference on Nonlinear Dynamics, School of Mathematics, University of New South Wales, Australia, "Computational Chaos in Integrable Systems", March 27-28, 1996.
19. Department of Mathematics, University of Sydney, Australia, "100 years of Integrability", April 3, 1996.
20. Workshop on Symmetries and Integrability, Kent University, Canterbury, England, "Solutions to the Time Dependent Schrödinger and the Kadomtsev-Petviashvili Equations", July 1–5, 1996.
21. Workshop in Nonlinear Optics, Mathematics Department, University of Arizona, Tucson, Arizona, "Wavelength Division Multiplexed Solitons and Four Wave Mixing", October 10–12, 1996.
22. International Symposium on Advances in soliton theory and its applications—The 30th anniversary of the Toda lattice, Yokohama National University, Yokohama, Japan, "On a New Class of Lump Type Solutions to the Kadomtsev-Petviashvili and Nonstationary Schrödinger Equations", December 1–4, 1996.
23. Conference on Soliton Theory, PDE's and Nonlinear Analysis, University of New South Wales, Sydney, Australia, "On Solutions of the Nonstationary Schrödinger and Kadomtsev-Petviashvili Equations", Jan. 6-9, 1997.
24. Conference on Chaos and Integrability in Discrete Systems, International Solvay Institutes for Physics and Chemistry, Vrije University Brussels, Brussels, Belgium, "Computational and Effective Chaos in Integrable Systems", July 1–5, 1997.

- **Consultative And Advisory Functions To Other Laboratories And Agencies**
- **Transitions**

NEW DISCOVERIES, INVENTIONS, OR PATENT DISCLOSURES

None

HONORS/AWARDS (M.J. Ablowitz)

1. Sloan Foundation Fellowship: 1975–1977
2. John Simon Guggenheim Fellowship: 1984–85
3. University of Colorado Council of Research and Creative Work Fellowship: 1994–95